Section 12.7: Conic Sections

Quick Review 12.7

1. The vertex of the parabola defined by $y = ax^2 + bx + c$ has an x-coordinate of $x = -\frac{b}{2a}$ and a y-coordinate of $f\left(-\frac{b}{2a}\right)$.

2. The x-coordinate of the vertex of $f(x) = 2x^2 + 4x + 3$ is

$$x = -\frac{b}{2a} = -\frac{4}{2(2)} = -1.$$

The y-coordinate is

$$f(-1) = 2(-1)^2 + 4(-1) + 3 = 2 - 4 + 3 =$$

Vertex: $(-1, 1)$

3.
$$\begin{aligned}
&9(x-2)^2 + 4(y+1)^2 - 36\\
&= 9(x^2 - 4x + 4) + 4(y^2 + 2y + 1) - 36\\
&= 9x^2 - 36x + 36 + 4y^2 + 8y + 4 - 36\\
&= 9x^2 + 4y^2 - 36x + 8y + 36 + 4 - 36\\
&= 9x^2 + 4y^2 - 36x + 8y + 4
\end{aligned}$$

4.
$$\begin{aligned}
2x^2 + 5x + 7 &= 0\\
\frac{2x^2}{2} + \frac{5x}{2} &= \frac{-7}{2}\\
x^2 + \frac{5}{2}x + \frac{25}{16} &= \frac{-7}{2} + \frac{25}{16}\\
\left(x + \frac{5}{4}\right)^2 &= -\frac{31}{16}\\
x + \frac{5}{4} &= \pm\sqrt{-\frac{31}{16}}\\
x &= -\frac{5}{4} \pm \frac{i\sqrt{31}}{4}\\
x &= \frac{-5 \pm i\sqrt{31}}{4} = \frac{-5}{4} \pm \frac{i\sqrt{31}}{4}
\end{aligned}$$

5.
$$\begin{aligned}
f(x) &= x^2 + 12x - 30\\
&= x^2 + 12x + 36 - 30 - 36\\
&= (x^2 + 12x + 36) - 30 - 36\\
&= (x+6)^2 - 66
\end{aligned}$$

Exercises 12.7

1. Distance: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(1-(-3))^2+((-1)-2)^2}$
$=\sqrt{(4)^2+(-3)^2}$
$=\sqrt{16+9}=\sqrt{25}=5$

Midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{(-3)+1}{2}, \frac{2+(-1)}{2}\right)$
$=\left(\frac{-2}{2}, \frac{1}{2}\right)=\left(-1, \frac{1}{2}\right)$

3. Distance: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(0-1)^2+(1-2)^2}$
$=\sqrt{(-1)^2+(-1)^2}$
$=\sqrt{1+1}=\sqrt{2}$

Midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{0+1}{2}, \frac{1+2}{2}\right)$
$=\left(\frac{1}{2}, \frac{3}{2}\right)$

5. Use the distance formula:
$r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(7-0)^2+(24-0)^2}$
$=\sqrt{(7)^2+(24)^2}=\sqrt{49+576}=25$

7. Use the midpoint formula:
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{(-1)+5}{2}, \frac{5+(-3)}{2}\right)$
$=\left(\frac{4}{2}, \frac{2}{2}\right)=(2, 1)$

9. The equation $x^2+y^2=4$ is in the standard form of a circle. Thus the graph is **B**.

11. The equation $\frac{x^2}{1}-\frac{y^2}{4}=1$ is in the standard form of a hyperbola. Thus the graph is **D**.

13. Substitute: $(h, k)=(0, 0)$ and $r=10$ in the standard form of the equation of a circle.
$(x-h)^2+(y-k)^2=r^2$
$(x-0)^2+(y-0)^2=10^2$
$x^2+y^2=100$

15. Substitute: $(h, k)=(2, 6)$ and $r=\sqrt{2}$ in the standard form of the equation of a circle.
$(x-h)^2+(y-k)^2=r^2$
$(x-2)^2+(y-6)^2=(\sqrt{2})^2$
$(x-2)^2+(y-6)^2=2$

17. Substitute: $(h, k)=\left(0, \frac{1}{2}\right)$ and $r=\frac{1}{2}$ in the standard form of the equation of a circle.
$(x-h)^2+(y-k)^2=r^2$
$(x-0)^2+\left(y-\frac{1}{2}\right)^2=\left(\frac{1}{2}\right)^2$
$x^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{4}$

19. Write the equation in the standard form of the equation of a circle.
$(x+5)^2+(y-4)^2=64$
$(x-(-5))^2+(y-4)^2=8^2$
The circle has a center $(-5, 4)$ and radius 8.

21. Write the equation in the standard form of the equation of a circle by completing the square.

$$x^2 + y^2 - 6x = 0$$

$$x^2 - 6x + 9 + y^2 = 0 + 9$$

$$(x-3)^2 + y^2 = 9$$

$$(x-3)^2 + (y-0)^2 = 3^2$$

The circle has a center $(3, 0)$ and radius 3.

23. Write the equation in the standard form of the equation of a circle by completing the square.

$$x^2 + y^2 - 2x + 10y + 22 = 0$$

$$x^2 - 2x + 1 + y^2 + 10y + 25 = -22 + 1 + 25$$

$$(x-1)^2 + (y+5)^2 = 4$$

$$(x-1)^2 + (y-(-5))^2 = 2^2$$

The circle has a center $(1, -5)$ and radius 2.

25. Write the equation in the standard form of the equation of an ellipse.

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

$$\frac{(x-0)^2}{36} + \frac{(y-0)^2}{16} = 1$$

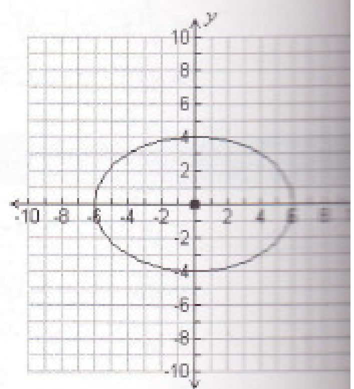
Center: $(0, 0)$

$a^2 = 36$ $\quad$ $b^2 = 16$

$a = 6$ $\quad$ $b = 4$

Length of major axis: $2a = 2(6) = 12$

Length of major axis: $2b = 2(4) = 8$



27. Write the equation in the standard form of the equation of an ellipse.

$$\frac{(y-3)^2}{25} + \frac{(x+4)^2}{16} = 1$$

$$\frac{(y-3)^2}{25} + \frac{(x-(-4))^2}{16} = 1$$

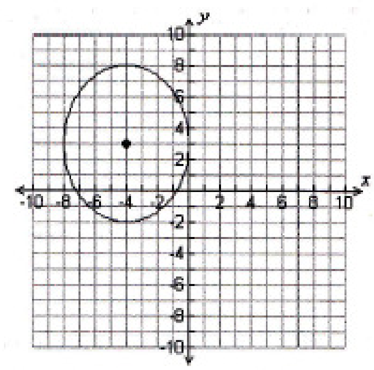
Center: $(-4, 3)$

$a^2 = 25$ $\quad$ $b^2 = 16$

$a = 5$ $\quad$ $b = 4$

Length of major axis: $2a = 2(5) = 10$

Length of major axis: $2b = 2(4) = 8$



29. $(h, k) = (0, 0)$

$a = 9; \quad b = 5$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-0)^2}{9^2} + \frac{(y-0)^2}{5^2} = 1$$

$$\frac{x^2}{81} + \frac{y^2}{25} = 1$$

31. $(h, k) = (3, 4)$

$a = \left(\frac{1}{2}\right)4 = 2; \quad b = \left(\frac{1}{2}\right)2 = 1$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-3)^2}{2^2} + \frac{(y-4)^2}{1^2} = 1$$

$$\frac{(x-3)^2}{4} + \frac{(y-4)^2}{1} = 1$$

33. $(h, k) = (6, -2)$

$a = \left(\frac{1}{2}\right)10 = 5; \quad b = \left(\frac{1}{2}\right)6 = 3$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-6)^2}{3^2} + \frac{(y-(-2))^2}{5^2} = 1$$

$$\frac{(x-6)^2}{9} + \frac{(y+2)^2}{25} = 1$$

35. $(h, k) = (-3, -4)$

$a = 6; \quad b = 2$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-(-3))^2}{2^2} + \frac{(y-(-4))^2}{6^2} = 1$$

$$\frac{(x+3)^2}{4} + \frac{(y+4)^2}{36} = 1$$

37. Write the equation in the standard form of the equation of a hyperbola.

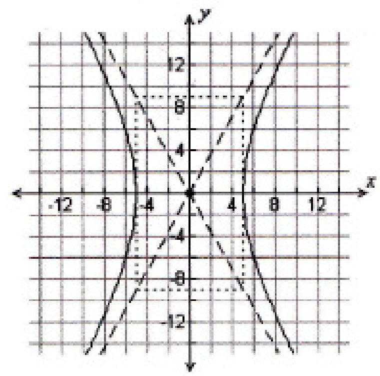
$$\frac{(x-0)^2}{25} - \frac{(y-0)^2}{81} = 1$$

Center: $(0, 0)$

$a^2 = 25 \qquad b^2 = 81$

$a = 5 \qquad b = 9$

The hyperbola opens horizontally.



39. Write the equation in the standard form of the equation of a hyperbola.

$$25x^2 = 100y^2 + 100$$

$$25x^2 - 100y^2 = 100$$

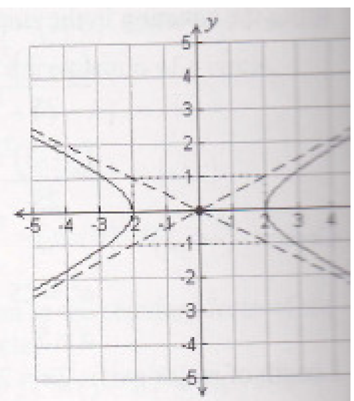
$$\frac{(x-0)^2}{4} - \frac{(y-0)^2}{1} = 1$$

Center: $(0, 0)$

$a^2 = 4 \qquad b^2 = 1$

$a = 2 \qquad b = 1$

The hyperbola opens horizontally.



41. Write the equation in the standard form of the equation of a hyperbola.

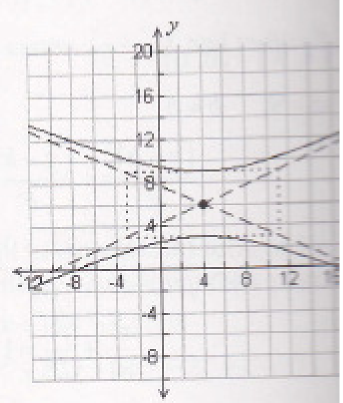
$$\frac{(y-6)^2}{9} - \frac{(x-4)^2}{49} = 1$$

Center: $(4, 6)$

$a^2 = 9 \qquad b^2 = 49$

$a = 3 \qquad b = 7$

The hyperbola opens vertically.



43. $(h, k) = (0, 0)$

$a = \left(\frac{1}{2}\right)10 = 5; \quad b = \left(\frac{1}{2}\right)8 = 4$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y-0)^2}{5^2} - \frac{(x-0)^2}{4^2} = 1$$

$$\frac{y^2}{25} - \frac{x^2}{16} = 1$$

45. $(h, k) = (0, 0)$

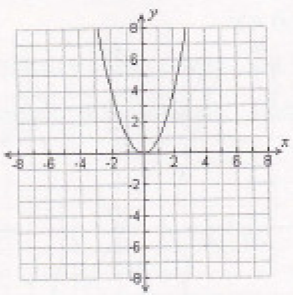
$a = 3; \quad b = \left(\frac{1}{2}\right)14 = 7$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

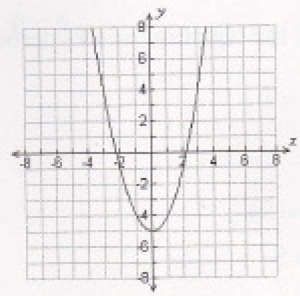
$$\frac{(x-0)^2}{3^2} - \frac{(y-0)^2}{7^2} = 1$$

$$\frac{x^2}{9} - \frac{y^2}{49} = 1$$

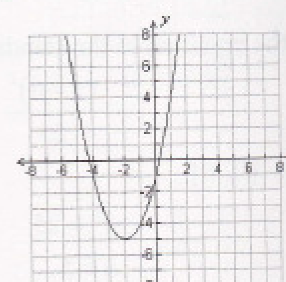
47. **a.**



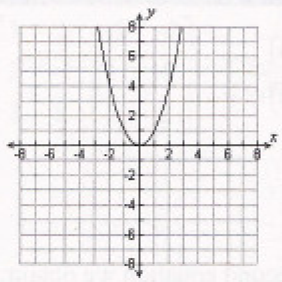
b.



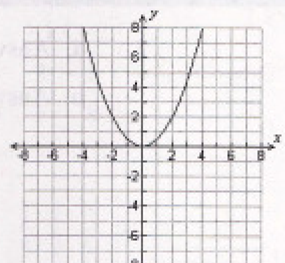
c.



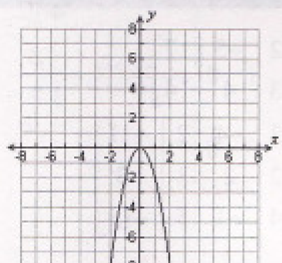
49. **a.**



b.

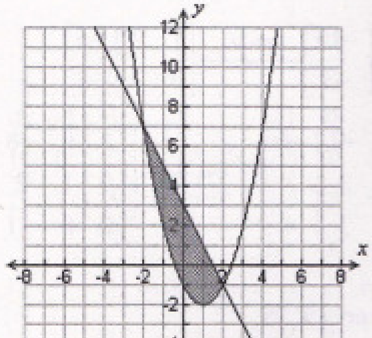


c.



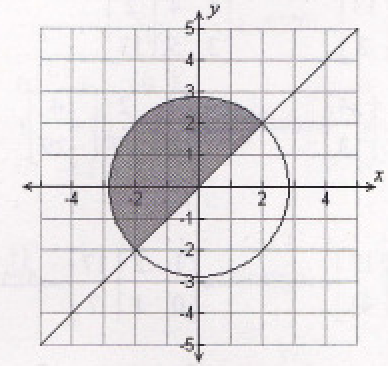
51. **a.** The solutions to the system of equations are the points of intersection: $(-2, 7)$ and $(2, -1)$.

b.



53. **a.** The solutions to the system of equations are the points of intersection: $(-2, -2)$ and $(2, 2)$.

b.



Cumulative Review

1.
$$3(2x-1)+5=7(4x-5)-34$$
$$6x-3+5=28x-35-34$$
$$6x+2=28x-69$$
$$-22x=-71$$
$$x=\frac{71}{22}$$

2. $|2x-1|=25$

is equivalent to

$2x-1=-25$ or $2x-1=25$

$2x=-24$ $\quad$ $2x=26$

$x=-12$ $\quad$ $x=13$

3.
$$(2x-1)^2=25$$
$$2x-1=\pm\sqrt{25}$$
$$2x-1=\pm 5$$
$$2x=1\pm 5$$
$$x=\frac{1\pm 5}{2}$$
$$x=-2 \text{ or } x=3$$

4.
$$\frac{3x+2}{2x-7}=4$$
$$\cancel{(2x-7)}\left(\frac{3x+2}{\cancel{2x-7}}\right)=(2x-7)4$$
$$3x+2=8x-28$$
$$-5x=-30$$
$$x=6$$

5.
$$\log(3x+1)=2$$
$$\log_{10}(3x+1)=2$$

is equivalent to $3x+1=10^2$
$$3x+1=100$$
$$3x=99$$
$$x=33$$