Quick Review 12.6

1. $a_5 = 23$

2. $a_n = 4n + 8$

3.

n	$a_n = 3n+5$
1	$3(1)+5=8$
2	$3(2)+5=11$
3	$3(3)+5=14$
4	$3(4)+5=17$
5	$3(5)+5=20$

4.

n	$a_n = n^2 - 5n + 8$
1	$(1)^2 - 5(1) + 8 = 4$
2	$(2)^2 - 5(2) + 8 = 2$
3	$(3)^2 - 5(3) + 8 = 2$
4	$(4)^2 - 5(4) + 8 = 4$
5	$(5)^2 - 5(5) + 8 = 8$

5.

n	$a_n = 5 \cdot 2^{n-1}$
1	$5 \cdot 2^{1-1} = 5 \cdot 2^0 = 5$
2	$5 \cdot 2^{2-1} = 5 \cdot 2^1 = 10$
3	$5 \cdot 2^{3-1} = 5 \cdot 2^2 = 20$
4	$5 \cdot 2^{4-1} = 5 \cdot 2^3 = 40$
5	$5 \cdot 2^{5-1} = 5 \cdot 2^4 = 80$

Exercises 12.6

1. **a.** The sequence is geometric. The common ratio is $r = \frac{6}{1} = \frac{36}{6} = \ldots = \frac{7776}{1296} = 6$.

b. The sequence is arithmetic. The common difference is $d = (6-1) = (11-5) = \ldots = (26-21) = 5$.

c. The sequence is neither arithmetic nor geometric. (The sequence does not have a common ratio or common difference.)

d. The sequence is arithmetic with a common difference of $d = (6-6) = (6-6) = \ldots = (6-6) = 0$.

It is also geometric with a common ratio of $r = \frac{6}{6} = \frac{6}{6} = \ldots = \frac{6}{6} = 1$.

3. **a.** The sequence: −2, 1, 4, 7, 10, 13,... is arithmetic.

The common difference is $d = (1-(-2)) = (4-1) = \ldots = (13-10) = \ldots = 3$.

b. The sequence: 1, 4, 9, 16, 25, 36,... is neither arithmetic nor geometric. (The sequence does not have a common ratio or common difference.)

c. The sequence: 4, 16, 64, 256, 1024,... is geometric.

The common ratio is $r = \frac{4}{1} = \frac{16}{4} = \ldots = \frac{1024}{256} = \ldots = 4$.

d. The sequence: 7, 7, 7, 7, 7,... is arithmetic with a common difference of

$d = (7-7) = (7-7) = \ldots = (7-7) = \ldots = 0$.

It is also geometric with a common ratio of $r = \frac{7}{7} = \frac{7}{7} = \ldots = \frac{7}{7} = \ldots = 1$.

5. **a.**

$a_1 = 5(1) + 3 = 8$

$a_2 = 5(2) + 3 = 13$

$a_3 = 5(3) + 3 = 18$

$a_4 = 5(4) + 3 = 23$

$a_5 = 5(5) + 3 = 28$

$a_6 = 5(6) + 3 = 33$

Answer: 8, 13, 18, 23, 28, 33,...

b.

$a_1 = (1)^2 - (1) + 1 = 1$

$a_2 = (2)^2 - (2) + 1 = 3$

$a_3 = (3)^2 - (3) + 1 = 7$

$a_4 = (4)^2 - (4) + 1 = 13$

$a_5 = (5)^2 - (5) + 1 = 21$

$a_6 = (6)^2 - (6) + 1 = 31$

Answer: 1, 3, 7, 13, 21, 31,...

c. $a_1 = 16\left(\frac{1}{2}\right)^1 = 8$

$a_2 = 16\left(\frac{1}{2}\right)^2 = 4$

$a_3 = 16\left(\frac{1}{2}\right)^3 = 2$

$a_4 = 16\left(\frac{1}{2}\right)^4 = 1$

$a_5 = 16\left(\frac{1}{2}\right)^5 = \frac{1}{2}$

$a_6 = 16\left(\frac{1}{2}\right)^6 = \frac{1}{4}$

Answer: $8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

d. $a_1 = 5(-2)^1 = -10$

$a_2 = 5(-2)^2 = 20$

$a_3 = 5(-2)^3 = -40$

$a_4 = 5(-2)^4 = 80$

$a_5 = 5(-2)^5 = -160$

$a_6 = 5(-2)^6 = 320$

Answer: $-10, 20, -40, 80, -160, 320\ldots$

7. **a.** $a_1 = 18$

$a_2 = 18 - 4 = 14$

$a_3 = 14 - 4 = 10$

$a_4 = 10 - 4 = 6$

$a_5 = 6 - 4 = 2$

$a_6 = 2 - 4 = -2$

Answer: $18, 14, 10, 6, 2, -2, \ldots$

b. $a_1 = 18$

$a_2 = 18 - 2 = 16$

$a_3 = 16 - 2 = 14$

$a_4 = 14 - 2 = 12$

$a_5 = 12 - 2 = 10$

$a_6 = 10 - 2 = 8$

Answer: $18, 16, 14, 12, 10, 8, \ldots$

c. $a_1 = 7$

$a_2 = 7 - 2 = 5$

$a_3 = 5 - 2 = 3$

$a_4 = 3 - 2 = 1$

$a_5 = 1 - 2 = -1$

$a_6 = -1 - 2 = -3$

Answer: $7, 5, 3, 1, -1, -3, \ldots$

d. $a_1 = -9$

$a_2 = -9 + 3 = -6$

$a_3 = -6 + 3 = -3$

$a_4 = -3 + 3 = 0$

$a_5 = 0 + 3 = 3$

$a_6 = 3 + 3 = 6$

Answer: $-9, -6, -3, 0, 3, 6, \ldots$

9. **a.** $a_1 = 1$

$a_2 = 4 \cdot 1 = 4$

$a_3 = 4 \cdot 4 = 16$

$a_4 = 4 \cdot 16 = 64$

$a_5 = 4 \cdot 64 = 256$

Answer: $1, 4, 16, 64, 256, \ldots$

b. $a_1 = 1$

$a_2 = 2 \cdot 1 = 2$

$a_3 = 2 \cdot 2 = 4$

$a_4 = 2 \cdot 4 = 8$

$a_5 = 2 \cdot 8 = 16$

Answer:
$1, 2, 4, 8, 16, \ldots$

or $a_1 = 1$

$a_2 = (-2) \cdot 1 = -2$

$a_3 = (-2) \cdot (-2) = 4$

$a_4 = (-2) \cdot 4 = -8$

$a_5 = (-2) \cdot (-8) = 16$

Answer:
$1, -2, 4, -8, 16, \ldots$

c. $a_1 = 16$

$a_2 = \frac{1}{2}(16) = 8$

$a_3 = \frac{1}{2}(8) = 4$

$a_4 = \frac{1}{2}(4) = 2$

$a_5 = \frac{1}{2}(2) = 1$

Answer: 16, 8, 4, 2, 1,...

d. $a_1 = 5$

$a_2 = -3 \cdot (5) = -15$

$a_3 = -3 \cdot (-15) = 45$

$a_4 = -3 \cdot (45) = -135$

$a_5 = -3 \cdot (-135) = 405$

Answer: 5, −15, 45, −135, 405,...

11. The n[th] term for an arithmetic sequence is determined by $a_n = a_1 + (n-1)d$.

a. $a_n = a_1 + (n-1)d$

$a_{83} = 6 + (83-1)5 = 416$

b. $d = 15 - 17 = -2$

$a_n = a_1 + (n-1)d$

$a_{51} = 17 + (51-1)(-2) = -83$

c. $d = 33 - 25 = 8$

$a_{82} = a_{81} + d$

$a_{82} = 33 + 8 = 41$

d. First, find *d*.

$a_3 = a_1 + (3-1)d$

$12 = 2 + 2d$

$2d = 10$

$d = 5$

$a_n = a_1 + (n-1)d$

$a_{101} = 2 + (101-1)(5) = 502$

13. $a_n = a_1 + (n-1)d$

$-62 = 18 + (n-1)(-4)$

$-80 = (n-1)(-4)$

$n - 1 = 20$

$n = 21$

15. $a_n = a_1 + (n-1)d$

$216 = a_1 + (44-1)(12)$

$216 = a_1 + 516$

$a_1 = -300$

17. $a_{31} = a_{11} + (20)d$

$14 = 4 + 20d$

$20d = 10$

$d = \frac{1}{2}$

19. $a_n = a_1 + (n-1)d$

$14 = 5 + (n-1)\left(\frac{1}{5}\right)$

$9 = (n-1)\left(\frac{1}{5}\right)$

$n - 1 = 45$

$n = 46$

21. $a_n = a_1 r^{n-1}$

$\frac{1}{3} = 243\left(\frac{1}{3}\right)^{n-1}$

$\frac{1}{729} = \left(\frac{1}{3}\right)^{n-1}$

$\left(\frac{1}{3}\right)^6 = \left(\frac{1}{3}\right)^{n-1}$

$n - 1 = 6$

$n = 7$

23. $a_n = a_1 r^{n-1}$

$24 = a_1 (2)^{5-1}$

$24 = a_1 (16)$

$a_1 = \frac{3}{2}$

25. $a_{11} = a_9 r^2$

$288 = 32(r)^2$

$r^2 = 9$

$r = 3 \text{ or } -3$

27. $a_n = a_1 r^{n-1}$

$a_9 = \left(\dfrac{1}{3125}\right)(5)^{9-1} = 125$

29. **a.** $\sum_{i=1}^{6}(2i+3) = (2(1)+3)+(2(2)+3)+(2(3)+3)+(2(4)+3)+(2(5)+3)+(2(6)+3)$

$= 5+7+9+11+13+15 = 60$

b. $\sum_{i=1}^{5}(i^2-1) = (1^2-1)+(2^2-1)+(3^2-1)+(4^2-1)+(5^2-1) = 0+3+8+15+24 = 50$

c. $\sum_{j=1}^{6}2^j = 2^1+2^2+2^3+2^4+2^5+2^6 = 2+4+8+16+32+64 = 126$

d. $\sum_{k=1}^{10}5 = 5+5+5+5+5+5+5+5+5+5 = 50$

$S_n = \dfrac{n}{2}(a_1 + a_n)$

$S_{40} = \dfrac{40}{2}(2+80) = 1{,}640$

33. $S_n = \dfrac{n}{2}(a_1 + a_n)$

$S_{12} = \dfrac{12}{2}\left(\dfrac{1}{2}+\dfrac{1}{3}\right) = 5$

$S_n = \dfrac{n}{2}(2a_1 + (n-1)d)$

$S_{66} = \dfrac{66}{2}(2(10)+(66-1)4) = 9{,}240$

37. $\sum_{i=1}^{61}(2i+3) = 5+7+9+11+\ldots+125$

$S_n = \dfrac{n}{2}(a_1 + a_n)$

$S_n = \dfrac{61}{2}(5+125) = 3{,}965$

$\sum_{k=1}^{24}\dfrac{k+3}{5} = \dfrac{4}{5}+\dfrac{5}{5}+\dfrac{6}{5}+\dfrac{7}{5}+\ldots+\dfrac{27}{5}$

$S_n = \dfrac{n}{2}(a_1 + a_n)$

$S_n = \dfrac{24}{2}\left(\dfrac{4}{5}+\dfrac{27}{5}\right) = \dfrac{372}{5}$

41. $S_n = \dfrac{a_1(1-r^n)}{1-r}$

$S_7 = \dfrac{3(1-2^7)}{1-2} = 381$

$S_n = \dfrac{a_1(1-r^n)}{1-r}$

$S_5 = \dfrac{0.2(1-0.1^5)}{1-0.1} = 0.22222$

45. $S_n = \dfrac{a_1(1-r^n)}{1-r}$

$S_8 = \dfrac{48\left(1-\left(-\dfrac{1}{2}\right)^8\right)}{1-\left(-\dfrac{1}{2}\right)} = \dfrac{255}{8}$

$S_n = \dfrac{a_1 - ra_n}{1-r}$

$S_n = \dfrac{1-(1.3)(3.71293)}{1-1.3} = 12.75603$

49. $S_n = \dfrac{a_1(1-r^n)}{1-r}$

$S_7 = \dfrac{0.8(1-0.1^7)}{1-0.1} = 0.8888888$

51. $S_n = \dfrac{a_1\left(1-r^n\right)}{1-r}$

$$S_6 = \dfrac{40\left(1-\left(\frac{1}{5}\right)^6\right)}{1-\left(\frac{1}{5}\right)} = \dfrac{31,248}{625}$$

53. Since $|r| < 1$ we may use the following form evaluate the infinite geometric series.

$$S = \dfrac{a_1}{1-r}$$

$$S = \dfrac{5}{1-\frac{2}{3}} = 15$$

55. Since $|r| < 1$ we may use the following formula to evaluate the infinite geometric series.

$$S = \dfrac{a_1}{1-r}$$

$$S = \dfrac{14}{1-\left(-\frac{3}{4}\right)} = 8$$

57. Since $|r| < 1$ we may use the following form evaluate the infinite geometric series.

$$S = \dfrac{a_1}{1-r}$$

$$S = \dfrac{0.12}{1-\frac{1}{100}} = \dfrac{4}{33}$$

59. Since $|r| < 1$ we may use the following formula to evaluate the infinite geometric series.

$$S = \dfrac{a_1}{1-r}$$

$$S = \dfrac{\left(\frac{4}{9}\right)}{1-\left(\frac{4}{9}\right)} = \dfrac{4}{5}$$

61. Since $|r| < 1$ we may use the following form evaluate the infinite geometric series.

$$S = \dfrac{a_1}{1-r}$$

$$S = \dfrac{6}{1-\left(-\frac{2}{3}\right)} = \dfrac{18}{5}$$

63. Since $|r| < 1$ we may use the following formula to evaluate the infinite geometric series.

$$S = \dfrac{a_1}{1-r}$$

$$S = \dfrac{24}{1-\frac{3}{8}} = \dfrac{192}{5}$$

65. **a.** $0.444... = 0.4 + 0.04 + 0.004 + ...$

$$= 4(0.1) + 4(0.1)^2 + 4(0.1)^3 + ...$$

$$S = \dfrac{a_1}{1-r} = \dfrac{0.4}{1-0.1} = \dfrac{4}{9}$$

b. $0.212121... = 0.21 + 0.0021 + 0.000021 + ...$

$$= 21(0.01) + 21(0.01)^2 + 21(0.01)^3$$

$$S = \dfrac{a_1}{1-r} = \dfrac{0.21}{1-0.01} = \dfrac{7}{33}$$

c. $0.409409... = 0.409 + 0.000409 + 0.000000409 + ...$

$$= 409(0.001) + 409(0.001)^2 + 409(0.001)^3 + ...$$

$$S = \dfrac{a_1}{1-r} = \dfrac{0.409}{1-0.001} = \dfrac{409}{999}$$

d. $2.5555... = 2 + 0.5 + 0.05 + 0.005 + ...$

$$= 2 + 5(0.1) + 5(0.1)^2 + 5(0.1)^3 + ...$$

$$S = 2 + \dfrac{a_1}{1-r} = 2 + \dfrac{.5}{1-0.1} = \dfrac{23}{9}$$

67. First determine a_1 by using the formula for the n[th] term of an arithmetic sequence.

$$a_n = a_1 + (n-1)d$$
$$19 = a_1 + (77-1)(-11)$$
$$19 = a_1 + (-836)$$
$$a_1 = 855$$

Now use the formula for the sum of the first n terms of an arithmetic sequence.

$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$S_{77} = \frac{77}{2}(855+19) = 33,649$$

69. $S_n = \frac{n}{2}(a_1 + a_n)$

$$1560 = \frac{30}{2}(a_1 + 93)$$
$$104 = a_1 + 93$$
$$a_1 = 11$$

71. $S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$$680 = \frac{40}{2}(2(11) + (40-1)d)$$
$$34 = 22 + 39d$$
$$39d = 12$$
$$d = \frac{4}{13}$$

73. $S_n = \frac{a_1(1-r^n)}{1-r}$

$$6138 = \frac{a_1(1-2^{10})}{1-2}$$
$$a_1 = 6138\left(\frac{-1}{1-2^{10}}\right)$$
$$a_1 = 6$$

75. $S_n = \frac{a_1 - ra_n}{1-r}$

$$32766 = \frac{6 - r(24576)}{1-r}$$
$$32766(1-r) = 6 - 24576r$$
$$32766 - 32766r = 6 - 24576r$$
$$-8190r = -32760$$
$$r = 4$$

77. $S = \frac{a_1}{1-r}$

$$21 = \frac{a_1}{1-\frac{2}{9}}$$
$$a_1 = 21\left(1-\frac{2}{9}\right)$$
$$a_1 = \frac{49}{3}$$

79. Find the following sum.

$$8+9+10+11+...+24$$

There are 17 terms in the arithmetic series. Thus the sum is

$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$S_{17} = \frac{17}{2}(8+24) = 272$$

There are 272 logs in the stack.

81. Given the following arithmetic series, find S_{12}.

$$4000 + 4500 + 5000 + ...$$
$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$
$$S_{12} = \frac{12}{2}(2(4000) + (12-1)500) = \$81,000$$

83. Given the following geometric series, find S_8.

$$1+4+16+64+\ldots$$

$$S_n = \frac{a_1\left(1-r^n\right)}{1-r}$$

$$S_8 = \frac{1\left(1-4^8\right)}{1-4} = 21{,}845 \text{ participants}$$

85. The terms in the following sequence give us percent of air that is left in the vessel after ea cycle.

$$100\left(\frac{2}{3}\right), 100\left(\frac{2}{3}\right)^2, 100\left(\frac{2}{3}\right)^3, \ldots$$

After the 8^{th} cycle, there will be $100\left(\frac{2}{3}\right)^8$ %

Thus $\left[100-100\left(\frac{2}{3}\right)^8\right]\% \approx 96.1\%$ of the air been removed.

87. Determine the following infinite sum.

$$600000\left(\frac{3}{4}\right)+600000\left(\frac{3}{4}\right)^2+600000\left(\frac{3}{4}\right)^2+\ldots$$

$$S = \frac{a_1}{1-r}$$

$$S = \frac{600000\left(\frac{3}{4}\right)}{1-\frac{3}{4}} = \$1{,}800{,}000$$

Cumulative Review

1. $12{,}000 = 1.2\times10^4$

2. $0.0045 = 4.5\times10^{-3}$

3. $-3 \le x < 5$

4. $x \le 2$

5. $[2, 7]$