**Quick Review 12.5**

1. $$\begin{aligned}(2x^2-7x-15)+(2x+3) &= 2x^2-7x-15+2x+3\\ &= 2x^2-7x+2x-15+3\\ &= 2x^2-5x-12\end{aligned}$$

2. $$\begin{aligned}(2x^2-7x-15)-(2x+3) &= 2x^2-7x-15-2x-3\\ &= 2x^2-7x-2x-15-3\\ &= 2x^2-9x-18\end{aligned}$$

3. $$\begin{aligned}&(2x^2-7x-15)(2x+3)\\ &= 4x^3+6x^2-14x^2-21x-30x-45\\ &= 4x^3-8x^2-51x-45\end{aligned}$$

4. $$\frac{2x^2-7x-15}{2x+3} = \frac{\cancel{(2x+3)}(x-5)}{\cancel{(2x+3)}} = x-5$$

5. First, write $y = 2x+3$.

Next, swap all values of $x$ and $y$.

$$x = 2y+3$$

Now, solve for $y$.

$$\begin{aligned}x &= 2y+3\\ x-3 &= 2y\\ y &= \frac{x-3}{2}\end{aligned}$$

The inverse of $f(x)$ is $f^{-1}(x) = \dfrac{x-3}{2}$.

## Exercises 12.5

**1.** **a.** $(f+g)(2)=f(2)+g(2)$
$$=\left((2)^2-1\right)+\left(2(2)+5\right)$$
$$=(3)+(9)=12$$

**b.** $(f-g)(2)=f(2)-g(2)$
$$=\left((2)^2-1\right)-\left(2(2)+5\right)$$
$$=(3)-(9)=-6$$

**c.** $(f\cdot g)(2)=f(2)\cdot g(2)$
$$=\left((2)^2-1\right)\cdot\left(2(2)+5\right)$$
$$=(3)\cdot(9)=27$$

**d.** $\left(\dfrac{f}{g}\right)(2)=\dfrac{f(2)}{g(2)}$
$$=\frac{\left((2)^2-1\right)}{\left(2(2)+5\right)}$$
$$=\frac{3}{9}=\frac{1}{3}$$

**3.**

| $x$ | $f+g$ |
|---|---|
| 0 | $(-2)+(-1)=-3$ |
| 3 | $(0)+(5)=5$ |
| 8 | $(7)+(9)=16$ |

**5.**

| $x$ | $g-f$ |
|---|---|
| 0 | $(-1)-(-2)=1$ |
| 3 | $(5)-(0)=5$ |
| 8 | $(9)-(7)=2$ |

**7.**

| $x$ | $g\cdot f$ |
|---|---|
| 0 | $(-1)\cdot(-2)=2$ |
| 3 | $(5)\cdot(0)=0$ |
| 8 | $(9)\cdot(7)=63$ |

**9.**

| $x$ | $f-g$ |
|---|---|
| $-2$ | $3-4=-1$ |
| 1 | $5-(-1)=6$ |
| 4 | $7-6=1$ |

$f-g=\{(-2,-1),(1,6),(4,1)\}$

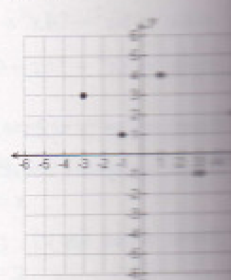
**11.**

| $x$ | $f+g$ |
|---|---|
| $-2$ | $\dfrac{3}{4}$ |
| 1 | $\dfrac{5}{-1}=-5$ |
| 4 | $\dfrac{7}{6}$ |

$f-g=\left\{\left(-2,\dfrac{3}{4}\right),(1,-5),\left(4,\dfrac{7}{6}\right)\right\}$

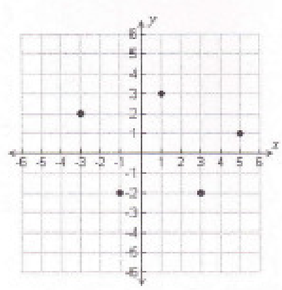
**13.**

| $x$ | $f+g$ |
|---|---|
| $-3$ | $1+2=3$ |
| $-1$ | $2+(-1)=1$ |
| 1 | $3+1=4$ |
| 3 | $(-2)+1=-1$ |
| 5 | $1+1=2$ |



**15.**

| $x$ | $f\cdot g$ |
|---|---|
| $-3$ | $1\cdot 2=2$ |
| $-1$ | $2\cdot(-1)=-2$ |
| 1 | $3\cdot 1=3$ |
| 3 | $(-2)\cdot 1=-2$ |
| 5 | $1\cdot 1=1$ |



**17.** $f$ and $g$ are not equal. $f(2)$ is defined and $g(2)$ is not defined. (The functions do not have the same domains.)

**21.** $f$ and $g$ are equal.

**23.** **a.** $(f \circ g)(2) = f(g(2))$
$= f(2(2)+5)$
$= f(9) = (9)^2 - 1 = 80$

**b.** $(g \circ f)(2) = g(f(2))$
$= g((2)^2 - 1)$
$= g(3) = 2(3) + 5 = 11$

**c.** $(f \circ f)(2) = f(f(2))$
$= f((2)^2 - 1)$
$= f(3) = 3^2 - 1 = 8$

**d.** $(g \circ g)(2) = g(g(2))$
$= g(2(2)+5)$
$= g(9) = 2(9) + 5 = 23$

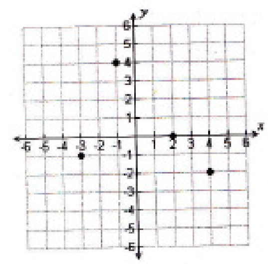
**25.**

| $x \xrightarrow{g} g(x) \xrightarrow{f} f(g(x))$ | $x \longrightarrow f(g(x))$ |
|---|---|
| $3 \longrightarrow 5 \longrightarrow 9$ | $3 \longrightarrow 9$ |
| $4 \longrightarrow 2 \longrightarrow 8$ | $4 \longrightarrow 8$ |
| $7 \longrightarrow 1 \longrightarrow 0$ | $7 \longrightarrow 0$ |
| $8 \longrightarrow 4 \longrightarrow 6$ | $8 \longrightarrow 6$ |

**27.** **a.**

| $x \xrightarrow{g} g(x) \xrightarrow{f} f(g(x))$ | $x \longrightarrow f(g(x))$ |
|---|---|
| $-3 \longrightarrow 2 \longrightarrow -1$ | $-3 \longrightarrow -1$ |
| $-1 \longrightarrow 3 \longrightarrow 4$ | $-1 \longrightarrow 4$ |
| $2 \longrightarrow -2 \longrightarrow 0$ | $2 \longrightarrow 0$ |
| $4 \longrightarrow 0 \longrightarrow -2$ | $4 \longrightarrow -2$ |

$(f \circ g)(x) = \{(-3,-1), (-1,4), (2,0), (4,-2)\}$



**29.** $(f \circ g)(x) = f(g(x)) = f(4x-2) = (4x-2)^2 - 5(4x-2) + 3 = 16x^2 - 16x + 4 - 20x + 10 + 3$
$= 16x^2 - 36x + 17$
$(g \circ f)(x) = g(f(x)) = g(x^2 - 5x + 3) = 4(x^2 - 5x + 3) - 2 = 4x^2 - 20x + 12 - 2 = 4x^2 - 20x + 10$

**31.** $(f \circ g)(x) = f(g(x)) = f(x+5) = \sqrt{x+5}$
$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{x} + 5$

**33.** $(f \circ g)(x) = f(g(x)) = f(x^2 - 4) = \dfrac{1}{(x^2-4)+2} = \dfrac{1}{x^2-2}$

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{1}{x+2}\right) = \left(\dfrac{1}{x+2}\right)^2 - 4 = \dfrac{1}{(x+2)^2} - 4 = \dfrac{1-4(x+2)^2}{(x+2)^2}$

$= \dfrac{1-4x^2-16x-16}{(x+2)^2} = -\dfrac{4x^2+16x+15}{(x+2)^2} = -\dfrac{(2x+3)(2x+5)}{(x+2)^2}$

**35.** $(f+g)(x)=(6x^2-x-15)+(3x-5)=6x^2-x-15+3x-5=6x^2+2x-20$; Domain: $(-\infty,\infty)$ or $\mathbb{R}$

$(f-g)(x)=(6x^2-x-15)-(3x-5)=6x^2-x-15-3x+5=6x^2-4x-10$; Domain: $(-\infty,\infty)$ or $\mathbb{R}$

$$(f\cdot g)(x)=(6x^2-x-15)\cdot(3x-5)=6x^2(3x-5)-x(3x-5)-15(3x-5)$$
$$=18x^3-30x^2-3x^2+5x-45x+75=18x^3-33x^2-40x+75;$$ Domain: $(-\infty,\infty)$ or $\mathbb{R}$

$$(f+g)(x)=\frac{6x^2-x-15}{3x-5}=\frac{\cancel{(3x-5)}(2x+3)}{\cancel{3x-5}}=2x+3;$$ Domain: $\left(-\infty,\frac{5}{3}\right)\cup\left(\frac{5}{3},\infty\right)$ or $\mathbb{R}\sim\left\{\frac{5}{3}\right\}$

$$(f\circ g)(x)=f(g(x))=f(3x-5)=6(3x-5)^2-(3x-5)-15=6(9x^2-30x+25)-3x+5-15$$
$$=54x^2-180x+150-3x-10=54x^2-183x+140;$$ Domain: $(-\infty,\infty)$ or $\mathbb{R}$

**37.** $(f+g)(x)=(5x-7)+(3)=5x-4$; Domain: $(-\infty,\infty)$ or $\mathbb{R}$

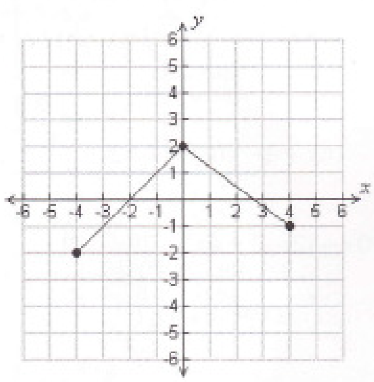
$(f-g)(x)=(5x-7)-(3)=5x-10$; Domain: $(-\infty,\infty)$ or $\mathbb{R}$

$(f\cdot g)(x)=(5x-7)\cdot(3)=15x-21$; Domain: $(-\infty,\infty)$ or $\mathbb{R}$

$\left(\frac{f}{g}\right)(x)=\frac{5x-7}{3}$; Domain: $(-\infty,\infty)$ or $\mathbb{R}$

$(f\circ g)(x)=f(g(x))=f(3)=5(3)-7=8$; Domain: $(-\infty,\infty)$ or $\mathbb{R}$

**39.**



**41.** $(f\circ f^{-1})(x)=f(f^{-1}(x))$

$$=f\left(\frac{3x+1}{2}\right)=\frac{\cancel{2}\left(\frac{3x+1}{\cancel{2}}\right)-1}{3}$$
$$=\frac{(3x+1)-1}{3}=\frac{\cancel{3}x}{\cancel{3}}=x$$

$(f^{-1}\circ f)(x)=f^{-1}(f(x))$

$$=f^{-1}\left(\frac{2x-1}{3}\right)=\frac{\cancel{3}\left(\frac{2x-1}{\cancel{3}}\right)+1}{2}$$
$$=\frac{(2x-1)+1}{2}=\frac{\cancel{2}x}{\cancel{2}}=x$$

**43.** $h(x)=(f+g)(x)=f(x)+g(x)$
$$=(x^2+1)+(\sqrt{x})=x^2+\sqrt{x}+1$$

**45.** $h(x)=(g\circ f)(x)=g(f(x))$
$$=g(x^2+1)=\sqrt{x^2+1}$$

**47.** Let $g(x) = x^2 + 4$ (the "inside" function) and $f(x) = \sqrt[3]{x}$ (the "outside" function)

**49.** Let $g(x) = x + 2$ (the "inside" function) and $f(x) = x^2 + 3x + 5$ (the "outside" function)

**51.**
$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= (4u^2 - 3u) - (10u + 25) \\ &= 4u^2 - 3u - 10u - 25 \\ &= 4u^2 - 13u - 25 \end{aligned}$$
$$P(10) = 4(10)^2 - 13(10) - 25 = \$245$$

**53.**
**a.** $V(b) = 10b$
**b.** $f(b) = 3000$
**c.** $C(b) = 10b + 3000$
**d.** $A(b) = \dfrac{C(b)}{b} = \dfrac{10b + 3000}{b}$
**e.** $R(b) = 12b$
**f.**
$$\begin{aligned} P(b) &= R(b) - C(b) \\ &= (12b) - (10b - 3000) \\ &= 2b - 3000 \end{aligned}$$
**g.** $P(1000) = 2(1000) - 3000 = -\$1000$
**h.** $P(1500) = 2(1500) - 3000 = \$0$
**i.** $P(2000) = 2(2000) - 3000 = \$1000$
**j.** From part **h** we can determine that the company breaks even when 1,500 units are produced and sold.

**55.**
**a.** $S(10) = 5(10) = 50$; The factory produces 50 sofas in 10 hours.
**b.** $C(50) = (50)^2 - 6(50) + 500 = 2700$; The cost of producing 50 sofas is \$2700.
**c.** $(C \circ S)(10) = C(S(10)) = C(50) = 2700$; The cost of operating the factory for 10 hours is \$2700.
**d.** $(C \circ S)(t) = C(S(t)) = C(5t) = (5t)^2 - 6(5t) + 500 = 25t^2 - 30t + 500$
**e.** $(C \circ S)(40) = 25(40)^2 - 30(40) + 500 = 39{,}300$; The cost of operating the factory for 40 hours is \$39,300.
**f.** $(C \circ S)(100) = 25(100)^2 - 30(100) + 500 = 247{,}500$; The cost of operating the factory for 100 hours is \$247,500.

**57.**
**a.** $r(s) = \dfrac{1}{2}s$
**b.** $A(r) = \pi r^2$
**c.**
$$\begin{aligned} (A \circ r)(s) &= A(r(s)) \\ &= A\left(\frac{1}{2}s\right) = \pi\left(\frac{1}{2}s\right)^2 = \frac{1}{4}\pi s^2 \end{aligned}$$

**59.**
**a.** $A = w^2$
**b.** $w = 44 - 2x$
**c.**
$$\begin{aligned} (A \circ w)(x) &= A(w(x)) \\ &= A(44 - 2x) = (44 - 2x)^2 \end{aligned}$$
**d.**
$$\begin{aligned} V &= (\text{height})(\text{Area of Base}) \\ &= x(44 - 2x)^2 \end{aligned}$$

## Cumulative Review

**1.** The additive identity is $0$.

**2.** The multiplicative identity is $1$.

**3.** The additive inverse of $6$ is $-6$.

**4.** The multiplicative inverse of $6$ is $\dfrac{1}{6}$.

**5.** The property that justifies rewriting $7x(3x-2) + 5(3x-2)$ as $(7x+5)(3x-2)$ is the distributive property of multiplication over addition.