Exercises 12.1

1. $\left[\begin{array}{cc|c} 3 & 1 & 0 \\ 2 & -1 & -5 \end{array}\right]$

3. $\left[\begin{array}{cc|c} 4 & 0 & 12 \\ 3 & 2 & 1 \end{array}\right]$

5. $\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -6 \end{array}\right]$

7. $\begin{Bmatrix} 2x+3y=2 \\ 4x-3y=1 \end{Bmatrix}$

9. $\begin{Bmatrix} 2x+y=1 \\ x+3y=0 \end{Bmatrix}$

11. $\begin{Bmatrix} x=7 \\ y=-8 \end{Bmatrix}$

13. $\left[\begin{array}{cc|c} 2 & 1 & 1 \\ 1 & 3 & 0 \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_2} \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 2 & 1 & 1 \end{array}\right]$

15. $\left[\begin{array}{cc|c} 1 & -3 & -2 \\ 2 & -5 & 4 \end{array}\right] \xrightarrow{r_2'=r_2-2r_1} \left[\begin{array}{cc|c} 1 & -3 & -2 \\ 0 & 1 & 8 \end{array}\right]$

17. $\left[\begin{array}{cc|c} 3 & -1 & 3 \\ 6 & 4 & -6 \end{array}\right] \xrightarrow{r_1'=\frac{1}{3}r_1} \left[\begin{array}{cc|c} 1 & -\frac{1}{3} & 1 \\ 6 & 4 & -6 \end{array}\right]$

19. $\left[\begin{array}{cc|c} 1 & 5 & 16 \\ 0 & 1 & 3 \end{array}\right] \xrightarrow{r_1'=r_1-5r_2} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \end{array}\right]$

21. Answer: $(-5, 9)$

23. The last row of the matrix implies $0=7$ which is a contradiction. Thus there is no solution.

25. Since the last row of the matrix gives us an identity, we have a dependent system of equations with an infinite number of solutions. Solving for x in the first equation we obtain: $x=5-3y$. Thus the general solution of the system is $(5-3y,\ y)$. For $y=0,\ y=1,$ and $y=-1$, we obtain the following particular solutions: $(5, 0), (2, 1),$ and $(8, -1)$.

27. $\left[\begin{array}{cc|c} 2 & 6 & 8 \\ 3 & 7 & 10 \end{array}\right] \xrightarrow{r_1'=\frac{1}{2}r_1} \left[\begin{array}{cc|c} 1 & 3 & 4 \\ 3 & 7 & 10 \end{array}\right]$

29. $\left[\begin{array}{cc|c} 1 & 5 & 8 \\ 3 & 2 & -2 \end{array}\right] \xrightarrow{r_2'=r_2-3r_1} \left[\begin{array}{cc|c} 1 & 5 & 8 \\ 0 & -13 & -26 \end{array}\right]$

31. $\left[\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 3 & -9 \end{array}\right] \xrightarrow{r_2'=\frac{1}{3}r_2} \left[\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 1 & -3 \end{array}\right]$

33. $\left[\begin{array}{cc|c} 1 & 2 & 6 \\ 0 & 1 & 1 \end{array}\right] \xrightarrow{r_1'=r_1-2r_2} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 1 \end{array}\right]$

35. $\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & 1 & -5 \end{array}\right] \xrightarrow{r_2'=r_2-2r_1} \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & -5 & -15 \end{array}\right] \xrightarrow{r_2'=-\frac{1}{5}r_2} \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 1 & 3 \end{array}\right] \xrightarrow{r_1'=r_1-3r_2} \left[\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & 3 \end{array}\right]$; Answer: $(-4, 3)$

37. $\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 3 & 7 & 7 \end{array}\right] \xrightarrow{r_2'=r_2-3r_1} \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & -2 & 4 \end{array}\right] \xrightarrow{r_2'=-\frac{1}{2}r_2} \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & 1 & -2 \end{array}\right] \xrightarrow{r_1'=r_1-3r_2} \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -2 \end{array}\right]$; Answer: $(7, -2)$

39. $\left[\begin{array}{cc|c} 2 & 5 & -4 \\ 4 & 3 & 6 \end{array}\right] \xrightarrow{r_1'=\frac{1}{2}r_1} \left[\begin{array}{cc|c} 1 & \frac{5}{2} & -2 \\ 4 & 3 & 6 \end{array}\right] \xrightarrow{r_2'=r_2-4r_1} \left[\begin{array}{cc|c} 1 & \frac{5}{2} & -2 \\ 0 & -7 & 14 \end{array}\right] \xrightarrow{r_2'=-\frac{1}{7}r_2} \left[\begin{array}{cc|c} 1 & \frac{5}{2} & -2 \\ 0 & 1 & -2 \end{array}\right]$

$\xrightarrow{r_1'=r_1-\frac{5}{2}r_2} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array}\right]$; Answer: $(3, -2)$

41. $\left[\begin{array}{cc|c} 4 & -9 & 5 \\ 3 & 12 & 10 \end{array}\right] \xrightarrow{r_1'=\frac{1}{4}r_1} \left[\begin{array}{cc|c} 1 & -\frac{9}{4} & \frac{5}{4} \\ 3 & 12 & 10 \end{array}\right] \xrightarrow{r_2'=r_2-3r_1} \left[\begin{array}{cc|c} 1 & -\frac{9}{4} & \frac{5}{4} \\ 0 & \frac{75}{4} & \frac{25}{4} \end{array}\right] \xrightarrow{r_2'=\frac{4}{75}r_2} \left[\begin{array}{cc|c} 1 & -\frac{9}{4} & \frac{5}{4} \\ 0 & 1 & \frac{1}{3} \end{array}\right]$

$\xrightarrow{r_1'=r_1+\frac{9}{4}r_2} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & \frac{1}{3} \end{array}\right]$; Answer: $\left(2, \frac{1}{3}\right)$

43. $\left[\begin{array}{cc|c} 3 & -1 & 2 \\ 2 & 1 & 6 \end{array}\right] \xrightarrow{r_1'=\frac{1}{3}r_1} \left[\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{2}{3} \\ 2 & 1 & 6 \end{array}\right] \xrightarrow{r_2'=r_2-2r_1} \left[\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & \frac{5}{3} & \frac{14}{3} \end{array}\right] \xrightarrow{r_2'=\frac{3}{5}r_2} \left[\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{14}{5} \end{array}\right]$

$\xrightarrow{r_1'=r_1+\frac{1}{3}r_2} \left[\begin{array}{cc|c} 1 & 0 & \frac{8}{5} \\ 0 & 1 & \frac{14}{5} \end{array}\right]$; Answer: $\left(\frac{8}{5}, \frac{14}{5}\right)$

45. $\left[\begin{array}{cc|c} 6 & 4 & 11 \\ 10 & 6 & 17 \end{array}\right] \xrightarrow{r_1'=\frac{1}{6}r_1} \left[\begin{array}{cc|c} 1 & \frac{2}{3} & \frac{11}{6} \\ 10 & 6 & 17 \end{array}\right] \xrightarrow{r_2'=r_2-10r_1} \left[\begin{array}{cc|c} 1 & \frac{2}{3} & \frac{11}{6} \\ 0 & -\frac{2}{3} & -\frac{4}{3} \end{array}\right] \xrightarrow{r_2'=-\frac{3}{2}r_2} \left[\begin{array}{cc|c} 1 & \frac{2}{3} & \frac{11}{6} \\ 0 & 1 & 2 \end{array}\right]$

$\xrightarrow{r_1'=r_1-\frac{2}{3}r_2} \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 2 \end{array}\right]$; Answer: $\left(\frac{1}{2}, 2\right)$

47. $\left[\begin{array}{cc|c} 3 & 4 & 7 \\ 6 & 8 & 10 \end{array}\right] \xrightarrow{r_1'=\frac{1}{3}r_1} \left[\begin{array}{cc|c} 1 & \frac{4}{3} & \frac{7}{3} \\ 6 & 8 & 10 \end{array}\right] \xrightarrow{r_2'=r_2-6r_1} \left[\begin{array}{cc|c} 1 & \frac{4}{3} & \frac{7}{3} \\ 0 & 0 & -4 \end{array}\right]$

The last row is a contradiction. Thus there is no solution.

49. $\left[\begin{array}{cc|c} 2 & -1 & 5 \\ 4 & -2 & 10 \end{array}\right] \xrightarrow{r_1'=\frac{1}{2}r_1} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{5}{2} \\ 4 & -2 & 10 \end{array}\right] \xrightarrow{r_2'=r_2-4r_1} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 0 \end{array}\right]$

Since the last row of the matrix gives us an identity, we have a dependent system of equations with an infinite number of solutions. Solving for x in the first equation we obtain: $x = \frac{5}{2} + \frac{1}{2}y$. Thus the general solution of the system is $\left(\frac{5}{2} + \frac{1}{2}y, y\right)$.

51. Let x and y be the two numbers. If their sum is 160 and their difference is 4, then to find the numbers we solve the following system.

$\left\{\begin{array}{l} x+y=160 \\ x-y=4 \end{array}\right\} \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 160 \\ 1 & -1 & 4 \end{array}\right] \xrightarrow{r_2'=r_2-r_1} \left[\begin{array}{cc|c} 1 & 1 & 160 \\ 0 & -2 & -156 \end{array}\right] \xrightarrow{r_2'=-\frac{1}{2}r_2} \left[\begin{array}{cc|c} 1 & 1 & 160 \\ 0 & 1 & 78 \end{array}\right] \xrightarrow{r_1'=r_1-r_2} \left[\begin{array}{cc|c} 1 & 0 & 82 \\ 0 & 1 & 78 \end{array}\right]$

Thus the two numbers are 82 and 78.

53. Let x and y be the two angles. If they are complementary and one angle is $32°$ larger than the other, [illegible] find the angles we solve the following system.

$$\begin{cases} x+y=90 \\ x-y=32 \end{cases} \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 90 \\ 1 & -1 & 32 \end{array}\right] \xrightarrow{r_2'=r_2-r_1} \left[\begin{array}{cc|c} 1 & 1 & 90 \\ 0 & -2 & -58 \end{array}\right] \xrightarrow{r_2'=-\frac{1}{2}r_2} \left[\begin{array}{cc|c} 1 & 1 & 90 \\ 0 & 1 & 29 \end{array}\right] \xrightarrow{r_1'=r_1-r_2} \left[\begin{array}{cc|c} 1 & 0 & 61 \\ 0 & 1 & 29 \end{array}\right]$$

Thus the two angles are $61°$ and $29°$.

55. Let x be the fixed cost and y be the variable cost per costume. To find these values, we solve the following system.

$$\begin{cases} x+20y=3200 \\ x+30y=4300 \end{cases} \rightarrow \left[\begin{array}{cc|c} 1 & 20 & 3200 \\ 1 & 30 & 4300 \end{array}\right] \xrightarrow{r_2'=r_2-r_1} \left[\begin{array}{cc|c} 1 & 20 & 3200 \\ 0 & 10 & 1100 \end{array}\right] \xrightarrow{r_2'=\frac{1}{10}r_2} \left[\begin{array}{cc|c} 1 & 20 & 3200 \\ 0 & 1 & 110 \end{array}\right] \xrightarrow{r_1'=r_1-20r_2}$$ [illegible]

Thus the fixed cost is \$1,000 and the variable cost is \$110 per costume.

57. Let x be the rate of the boat and y be the rate of the current. To find these values, solve the following system.

$$\begin{cases} x+y=30 \\ x-y=14 \end{cases} \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 30 \\ 1 & -1 & 14 \end{array}\right] \xrightarrow{r_2'=r_2-r_1} \left[\begin{array}{cc|c} 1 & 1 & 30 \\ 0 & -2 & -16 \end{array}\right] \xrightarrow{r_2'=-\frac{1}{2}r_2} \left[\begin{array}{cc|c} 1 & 1 & 30 \\ 0 & 1 & 8 \end{array}\right] \xrightarrow{r_1'=r_1-r_2} \left[\begin{array}{cc|c} 1 & 0 & 22 \\ 0 & 1 & 8 \end{array}\right]$$

Thus the speed of the boat is 22 km/hr and the speed of the current is 8 km/hr.

59. Let x be amount of the fruit concentrate and y be the amount of pure water used. To find these values, solve the following system.

$$\begin{cases} x+y=100 \\ .15x+(1)y=100(.83) \end{cases} \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 100 \\ .15 & 1 & 83 \end{array}\right] \xrightarrow{r_2'=r_2-.15r_1} \left[\begin{array}{cc|c} 1 & 1 & 100 \\ 0 & .85 & 68 \end{array}\right] \xrightarrow{r_2'=\frac{1}{.85}r_2} \left[\begin{array}{cc|c} 1 & 1 & 100 \\ 0 & 1 & 80 \end{array}\right]$$

$$\xrightarrow{r_1'=r_1-r_2} \left[\begin{array}{cc|c} 1 & 0 & 20 \\ 0 & 1 & 80 \end{array}\right]$$

Thus they should use 20 liters of the fruit concentrate and 80 liters of pure water.

61. If the inside wheel covers 111.5 inches per revolution, then the circumference of the inside wheel is 111.5 inches and $2\pi r_1 = 111.5$. Thus $r_1 = \dfrac{111.5}{2\pi} \approx 17.75$ inches. The radius of the outside wheel is $r_2 = 17.75 + .25 = 18$ inches.

Cumulative Review

1.

n	$a_n = 2n+1$
1	$2(1)+1=3$
2	$2(2)+1=5$
3	$2(3)+1=7$
4	$2(4)+1=9$
5	$2(5)+1=11$

2. $a_{50} = 2(50)+1 = 101$

3.

n	$a_n = 2^{n+1}$
1	$2^{1+1}=2^2=4$
2	$2^{2+1}=2^3=8$
3	$2^{3+1}=2^4=16$
4	$2^{4+1}=2^5=32$
5	$2^{5+1}=2^6=64$

4. $(12a+3b)^0+(12a)^0+(3b)^0+12a^0+3b^0$
$=1+1+1+12(1)+3(1)=18$

5. $(-1)^4+4^{-1}+(1)^{1/4} = 1+\frac{1}{4}+\sqrt[4]{1} = 1+\frac{1}{4}+1$
$=2+\frac{1}{4}=\frac{2}{1}\cdot\frac{4}{4}+\frac{1}{4}=$ [illegible]